DBI global strings

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ABSTRACT: In this note we present global string solutions which are a generalization of the usual field theory global vortices when the kinetic term is DBI. Such vortices can result from the spontaneous symmetry breaking in the potential felt by a D3-brane. In a previous paper (hep-th/0706.0485), the DBI instanton solution was constructed which develops a "wrinkle" for stringy heights of the potential. A similar effect is also seen for the DBI vortex solution. The wrinkle develops for stringy heights of the potential. One recovers the usual field theory global string for substringy potentials. As an example of the symmetry breaking, we consider a mobile D3-brane on the warped deformed conifold. Symmetry breaking can occur if the structure of the vacuum manifold of the potential for the D3-brane changes as it moves through the throat region.

KEYWORDS: Solitons Monopoles and Instantons, String theory and cosmic strings.

Contents

1. Introduction

String theory embeddings of inflationary scenarios typically involve D-branes which are either static or undergoing some motion in the transverse directions. The world volume theory of the D-brane is described by the DBI action. Since the DBI action is a higher dimensional generalization of the Lorentz invariant relativistic action for a point particle, relativistic effects are to be expected for the D-brane dynamics under appropriate relativistic conditions. Such examples of DBI relativistic effect are studied in [1–4]. In [3], the speed limit arising due to the relativistic nature of the DBI action is responsible for inflation. In [4] a variant of this scenario was proposed, again based on the DBI action. Further, the DBI effect has been shown to prevent the occurence of slow roll eternal inflation in DBI inflation scenarios [5]. These inflationary scenarios employ mobile branes and, therefore, when the D3-brane speed becomes relativistic see DBI effects. However, the nonlinear nature of the DBI action can lead to new behavior in certain phenomena even when the D-brane is static. In [1] the tunneling of a D3-brane from a metastable to a true vacuum was investigated along the lines of the usual quantum field theory analysis given in [6]. For substringy barriers the tunneling picture for the DBI action matches with the usual QFT picture given in [6]. But once the barrier between the metastable and the true vacuum assumes stringy heights, tunneling is much more enhanced than what one would have expected based on the usual QFT intuition. This is surprising as the usual

QFT intuition has taught us to expect an exponential suppression in the tunneling probability with an increase in the height of the barrier. This counterintuitive result is easily understood, however, based on a similar treatement of a point charged particle tunneling through an electrostatic barrier [2]. Once the height of the barrier is large enough, there is a significant Schwinger pair production of point particle- antiparticle and this Schwinger effect can enhance tunneling. Similar effect for the D3-brane enhances the tunneling rate. The D-brane, therefore, need not be moving at relativistic speeds to see interesting DBI effects. In this paper we investigate global vortex solutions in the world volume of a D3-brane described by the DBI action. Similar to the findings in [1], these vortex solutions display a departure from the usual field theory behavior once the height of the potential of the complex scalar field becomes stringy.

The DBI vortex solutions that we will construct should not be confused with the vortex solutions of the tachyon field formed due to tachyon condensation after brane-antibrane annihilation (constructed, for example, in [15]) which correspond to codimension two branes. The DBI vortices we construct are generalizations of the well known field theory global vortices when the kinetic term is DBI. Indeed, for small gradients (which will correspond to small heights of the potential) the DBI vortex resembles the usual field theory vortex. Further, these vortices can form even when the separation between a brane and an antibrane is much greater than the critical distance when tachyon condensation initiates. For the tachyon vortices to form, the brane-antibrane separation should be string length.

2. The set-up

The set up involves a $D3$-brane on a warped background. The DBI action for the $D3$-brane is given by [3]

$$S = - \int d^4x \sqrt{-g} \left(f(\phi)^{-1} \sqrt{1 + f(\phi) g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi} - f(\phi)^{-1} + V(\phi) \right) \qquad (2.1)$$

where $f(\phi)$ is the warp factor. When the gradient terms are small, one can expand the square root and keep the lowest order gradient term, the above action then reduces to the usual field theory action with the quadratic kinetic term. We are interested in vortex solutions of the DBI action.

To study vortex solutions we must generalize the above action to the case of a complex scalar field $\psi = \psi_1 + i\psi_2$. We consider the action

$$S = - \int d^4x \sqrt{-g} \left(\sqrt{1 + g^{\mu\nu} \partial_\mu \psi^* \partial_\nu \psi} - 1 + V(|\psi|) \right) \qquad (2.2)$$

where $|\psi| = \sqrt{\psi \psi^*}$ is the absolute value. $V(\psi)$ has the mexican hat shape, $V(|\psi|) = V_0 (\psi^* \psi - 1)^2$. Note that ψ does not have to be the radial coordinate of eq. (2.1). ψ_1 and ψ_2 could be the angular coordinates at a fixed radial distance along the throat region of the warped deformed conifold.

As an example, consider the warped compactification of type IIB string theory down to four dimensions with metric ansatz

$$ds^2 = e^{2A(y)}g_{\mu\nu}dx^\mu dx^\nu + e^{-2A(y)}\tilde{g}_{mn}dy^m dy^n, \qquad (2.3)$$

where $e^{A(y)}$ is the warp factor related to $f(\phi)$ in eq. (2.1) by the relation $e^{4A(y)} = \alpha'^2/f(\phi)$. We consider compactifications which are of the GKP type, which includes the Klebanov-Strassler warped throat arising as part of a compact geometry. One begins with a conifold singularity with three-cycles A and B and complex structure modulus $\epsilon^2 = \int_A \Omega$, Ω being the holomorphic three form. ϵ is stabilized by the fluxes

$$\int_A F_3 = M,$$
$$\int_B H_3 = -K, \qquad (2.4)$$

at an exponentially small value $\epsilon \sim \exp(-\frac{\pi K}{g_s M})$. The resulting geometry is well described by a KS throat, over which the warp factor is strongly varying, attached to the rest of the compact space. The unwarped metric of this region is that of a deformed conifold which comes to a smooth end at the tip of the throat. Far from the tip, but still in the throat, the unwarped metric is given by

$$\tilde{g}_{mn}dy^m dy^n \simeq d\rho^2 + \rho^2 ds^2_{T^{1,1}}, \qquad (2.5)$$

$ds^2_{T^{1,1}}$ is the canonical metric on the five dimensional Einstein space $T^{1,1}$, which is topologically $S^3 \times S^2$. At the tip of the throat, the S^2 shrinks to zero size and the metric is well approximated by

$$\tilde{g}_{mn}dy^m dy^n \simeq \epsilon^{4/3}\left(d\tau^2 + \tau^2 d\Omega_2^2 + d\Omega_3^2\right), \qquad (2.6)$$

where $\rho^3 = \epsilon^2 cosh(\tau)$. At the tip $\tau = 0$.

In eq. (2.2), ψ_1 and ψ_2 could, for example, correspond to two angular directions in the S^3 at a fixed value of ρ. Then $V(|\psi|)$ then corresponds to the potential for these angular locations of the D3-brane. In particular, the moduli space of a D3-brane/antibrane arising due to nonperturbative Kahler moduli fixing effects has been constructed in [7]. The potential felt by the D3-brane/antibrane along angular directions of the S^3 of the warped deformed conifold has been explicitly constructed. Our choice of ψ_1 and ψ_2 corresponding to two angular coordinates on the S^3 is motivated by this construction.

3. Review of field theory global vortices

When the $(\partial_\mu \psi)^2$ term is small, the action (eq. (2.2)) reduces to the usual field theory action for a complex scalar field

$$S = \int d^4x \sqrt{-g}\left(\frac{1}{2}g^{\mu\nu}\partial_\mu\psi^*\partial_\nu\psi - V_0\left(\psi^*\psi - 1\right)^2\right). \qquad (3.1)$$

The equation of motion for ϕ is

$$\partial_\mu \partial^\mu \psi + 2V_0(|\psi|^2 - 1)\psi = 0. \tag{3.2}$$

One looks for a static axisymmetric solutions of the form

$$\psi(r) = e^{in\theta}\chi(r), \tag{3.3}$$

$\chi(r)$ satisfies the following second order nonlinear differential equation

$$\frac{d^2\chi}{dr^2} + \frac{1}{r}\frac{d\chi}{dr} - \frac{n^2}{r^2}\chi - 2V_0\chi(\chi^2 - 1) = 0. \tag{3.4}$$

One requires that $\chi(r)$ satisfy the following boundary conditions

$$\chi(r) \to 1, \quad r \to \infty,$$
$$\chi(0) = 0. \tag{3.5}$$

The resulting solution is a global vortex with the asymptotics

$$\chi(r) \approx c_n r^n + \dots, \qquad r \to 0,$$
$$\chi(r) \approx 1 - O(r^{-2}), \qquad r \to \infty. \tag{3.6}$$

The global vortex therefore corresponds to a static field configuration that sits on the vacuum locus $\phi^*\phi = 1$ at infinity and at the top of the potential at the core of the vortex (i.e. at $r = 0$). Using numerical methods one can further determine the coefficients c_n. For example, setting $V_0 = 0.5$, for $n = 1$ winding, one gets $c_1 \approx 0.58$. The energy per unit length of the global vortex solution is logarithmically divergent.

4. DBI global vortex solution

We generalize the above analysis to the DBI action (eq. (2.2)). Using the ansatz for axisymmetric solutions $\psi = e^{in\theta}\chi(r)$, the action (eq. (2.2)) reduces to

$$S = \int dr\, r \left[-\sqrt{1 + \left(\frac{d\chi}{dr}\right)^2 + \frac{n^2\chi^2}{r^2}} + 1 - V_0\left(\chi^2 - 1\right)^2 \right]. \tag{4.1}$$

The corresponding equation of motion for $\chi(r)$ is

$$\frac{d}{dr}\left(\gamma\frac{d\chi}{dr}\right) + \frac{\gamma}{r}\frac{d\chi}{dr} - \gamma\frac{n^2\chi}{r^2} - 2V_0\chi\left(\chi^2 - 1\right) = 0, \tag{4.2}$$

where the relativistic γ factor is given by

$$\gamma = \frac{1}{\sqrt{1 + \left(\frac{d\chi}{dr}\right)^2 + \frac{n^2\chi^2}{r^2}}}. \tag{4.3}$$

Eq. (4.2) is the DBI generalization of eq. (3.4). We would like to construct vortex solutions to this equation, i.e. solutions satisfying the boundary conditions $\chi(r = 0) = 0$, $\chi(r = \infty) = 1$. However, as we will see, the DBI action will present us with difficulties while trying to impose one boundary condition at $r = 0$ and one at infinity. Instead, we will be content with the (equivalent) boundary conditions at infinity $\chi(r = \infty) = 1$, $d\chi(r = \infty)/dr = 0$.

Analytic solutions even for the relatively simple eq. (3.4) are not known, numerical methods have to be employed. Therefore, we shall simply try and find numerical solutions to the much more complicated eq. (4.2). We construct numerical solutions where $\chi(r \to \infty) = 1$ and $d\chi(r \to \infty)/dr = 0$.

4.1 Summary of the plots

Before displaying the numerical results we summarize our findings. We focus on winding number 1, i.e. $n = 1$. We observe a phenomenon similar to that observed in the instanton context in reference [1]. Starting with a small substringy value for V_0 (height of the mexican hat potential at $\chi = 0$), we increase its value towards stringy values. In the numerical plots, for $V_0 < 0.8$ we see the usual global vortex solution, i.e. $\chi = 1$ at large r and $\chi = 0$ at the origin $r = 0$. At the origin, just like in usual field theory, $d\chi/dr$ has some finite value.

At $V_0 = 0.8$ (within the accuracy of these mathematica plots) we observe $d\chi/dr \to \infty$ close to (but not at) the origin. Further, the vortex configuration fails to go all the way to the origin. $d\chi/dr$ becomes very large at around $r = 0.1$. This implies a "turning around" is beginning to happen at this stage (similar to the "wrinkling" effect found in [1]).

The plot for $V_0 = 0.9$ shows a "turning around" of the vortex configuration, the field modulus χ has become double-valued near the origin. At infinity $\chi = 1$ as required for the vortex solution. As the radial distance r decreases the field χ also drops towards $\chi = 0$. However, because of the DBI effect, before the field reaches the origin $r = 0$, the gradient $d\chi/dr$ becomes infinity. This implies a "turning around" of the field configuration. Now r, instead of decreasing anymore, increases and χ keeps going towards $\chi = 0$. At $\chi = 0$, $d\chi/dr$ has some finite value and r is finite. This configuration (taking into account the radial symmetry) looks like a "throat".

Higher values of V_0 (plot for $V_0 = 1.5$) show the same behavior. The usual field theory vortex has blown up into a throat solution.

For $V_0 = 2.0$ we see that at $\chi = 0$, $d\chi/dr \to -0$. So the solution at $r = \infty$ has $d\chi/dr = +0$ (and $\chi = 1$). As r decreases, there is a turning around. After turning around the solution attains $d\chi/dr = -0$ when $\chi = 0$. This implies that the D-brane turns around , via the throat, into an antibrane. Presumably such a configuration will be unstable.

4.2 Plots

The mathematica plots of r (the radial distance) versus χ are shown in figures 1–8.

5. The D3-brane moduli space and formation of vortices

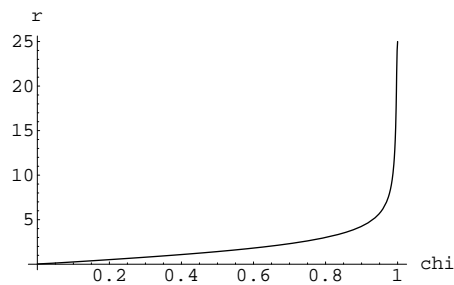In [7], D3-brane and antibrane vacua on the warped deformed conifold were constructed.

Figure 1: DBI global vortex solution with $V_0 = 0.1$. The vortex resembles a usual field theory global vortex.
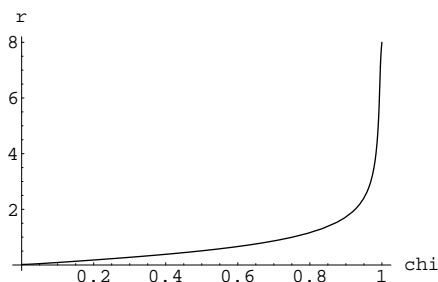

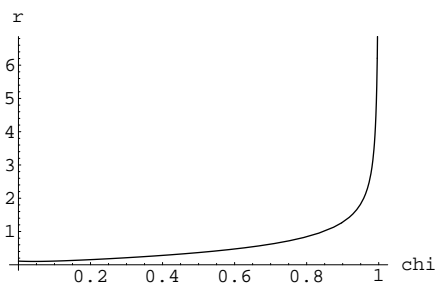**Figure 2:** DBI global vortex solution with $V_0 = 0.5$. The vortex resembles a usual field theory global vortex.

Figure 3: DBI global vortex solution with $V_0 = 0.8$. The gradient $d\chi/dr \to \infty$ at $r = 0.15$ and the field χ fails to reach the origin $r = 0$. At this value of V_0 the DBI effect is siginificant.

D3-branes do not feel any force in no-scale flux compactifications of type II-B string theory. However, the non-perturbative effects required to stabilize the Kahler moduli destroy the no-scale feature and generate a potential for the D3-brane. The shape of the potential depends on the compactification geometry and on the embedding of the moduli-stabilizing branes. In [7], various supersymmetric embeddings of the D7-brane have been considered in the context of the warped deformed conifold. These embeddings of the D7-brane generate a non-perturbative superpotential which in turn generates the D3-brane vacua. The vacua have real dimensions zero, one and two. The zero and one dimensional vacua are generic
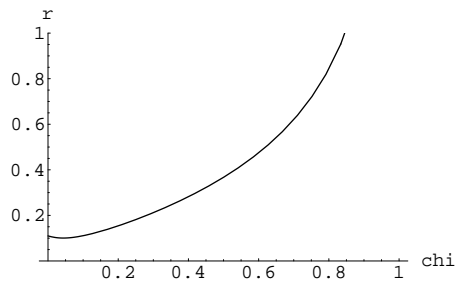
Figure 4: Once again DBI global vortex solution with $V_0 = 0.8$, but with the Y-axis range changed to show the "turning around" clearly.
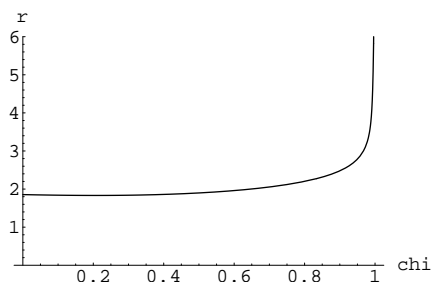

Figure 5: The DBI vortex solution with $V_0 = 0.9$. $d\chi/dr \to 0$ at large r, $d\chi/dr \to \infty$ at $r \approx 1.8$, then r increases again and $d\chi/dr$ becomes finite and negative as $\chi \to 0$.
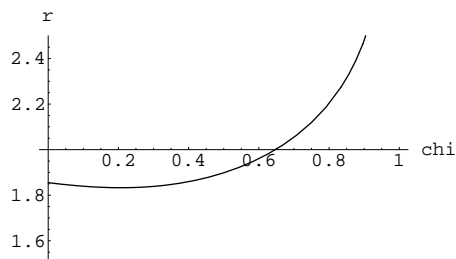

Figure 6: Once again DBI vortex solution with $V_0 = 0.9$. Y-axis range changed.

in a compact Calabi-Yau. Furthermore, the D3-branes and D3-antibranes share the same vacua at the tip of the warped deformed conifold (which is important for the success ending of brane-antibrane inflation). The authors give explicit moduli space of the vacua by considering the warped deformed conifold. At a fixed radial coordinate the deformed conifold has a $S^2 \times S^3$. The S^2 shrinks to a zero size at the tip. The zero, one and two dimensional vacua correspond to a point, S^1 and T^2 on the S^3.

The DBI vortex solutions constructed from the action in eq. (2.2), require the vacua to constitute a periodic one-dimensional loci (i.e. a S^1). This potential is modeled by the mexican hat potential of eq. (2.2).
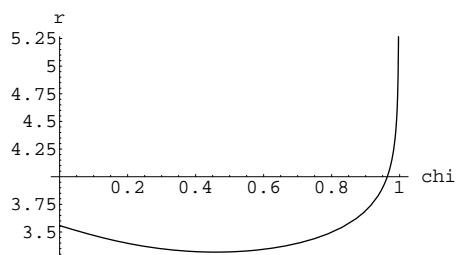
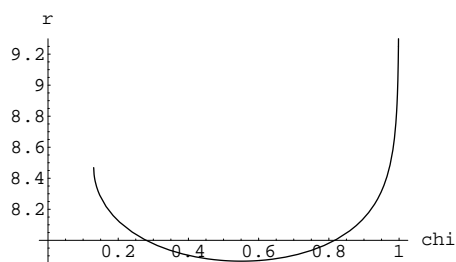**Figure 7:** DBI global vortex solution with $V_0 = 1.5$.

Figure 8: DBI global vortex solution with $V_0 = 2.0$. $d\chi/dr = -0$ as $\chi = 0$ at the bottom of the throat. In other words, the D-brane turns around and becomes an antibrane.

The particular symmetry breaking mechanism that we have in mind is depicted in figure 9. As an example of the symmetry breaking, we consider a mobile D3-brane on the warped deformed conifold. Spontaneous symmetry breaking can occur if the structure of the vacuum manifold of the potential experienced by the D3-brane changes as it moves through the throat region. We consider two angular directions ψ_1 and ψ_2 transverse to the D3-brane world volume (ψ^1 and ψ^2 are some coordinates on the S^3, for example). Together they constitute a complex scalar field $\psi = \psi_1 + i\psi_2$. The D3-brane potential $V(\psi)$ is a function of these two transverse directions. Apart from these two directions ψ_1 and ψ_2, the usual radial direction ρ along the warped deformed conifold is also present. The other transverse directions are not involved in the dynamics and are suppressed. We assume that away from the tip (nonzero radial coordinate ρ), the vacuum is zero dimensional (i.e. there is a unique minimum) and at the tip ($\rho = 0$) the vacua constitute an S^1. I.e., the potential has the form $V(\psi) = V_0 + m(\rho)^2|\psi|^2 + \lambda|\psi|^4$, with $m(\rho)^2 > 0$ away from the tip, and $m(\rho = 0)^2 < 0$ at the tip. The spontaneous symmetry breaking occurs as the D3-brane moves down the throat region towards the tip. Away from the tip the D3-brane sees a unique vacuum and at the tip it can settle down to any point of the S^1. From the D3-brane world volume perspective, the dynamics of the complex field ψ is described by the four dimensional effective action of eq. (2.2). If we replace the DBI kinetic term by the usual quadratic kinetic term $1/2\partial_\mu\psi\partial^\mu\psi^*$, then we are just looking at the usual second order phase transition of a complex $\lambda|\psi|^4$ theory that can form vortices. The radial
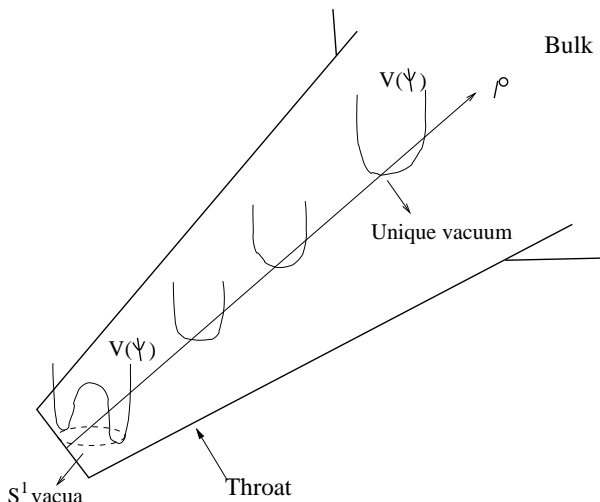
Figure 9: The throat region of a warped deformed conifold. ψ_1 and ψ_2 are two angular coordinates on the S^3 and together they constitute the complex scalar field ψ. The D3-brane feels the potential $V(\psi) = m(\rho)^2|\psi|^2 + \lambda|\psi|^4$. The ρ coordinate represents the radial direction along the throat, and is assumed to be a flat direction. $m(\rho)^2 > 0$ away from the tip and $m(\rho^2) < 0$ at the tip. Away from the tip, $V(\psi)$ has a unique vacuum. At the tip, the degenerate vacua of $V(\psi)$ constitute a $S1$. The spontaneous symmetry breaking occurs as the D3-brane moves towards and eventually sits at the tip.

coordinate ρ serves as the symmetry breaking parameter that controls the shape of the potential.

Points that are far away in the D3-brane world volume will be uncorrelated regarding their choice of the particular point on the S^1 to which they eventually settle down (the correlation length being the inverse mass scale $1/m$). Hence once the brane settles down at the tip, its world volume will consist of various domains with different values of the complex field ψ (all belonging to the S^1 moduli space). As a result vortices will form.

Although we have been referring to the vortex formation due to the D3-brane dynamics, the same applies to D3-antibrane as well. As discussed in [7], one can construct the moduli space for the antibrane on the warped deformed conifold and just like the D3-brane case, the corresponding vacua have real dimensions zero, one and two. At the tip of the warped deformed conifold, the D3-brane and the D3-antibrane share the same vacua. This is important for ending brane inflation. In brane inflation scenarios, the antibrane sits at the tip of the warped deformed conifold while the D3 brane falls towards the tip due to the attraction it feels from the antibrane. Can we apply the above spontaneous symmetry breaking mechanism to brane-antibrane inflation and produce DBI strings at the end of inflation? The antibrane will first feel the attraction from the tip region and migrate towards the tip. Assuming the presence of S^1 vacua at the tip, the antibrane will not prefer any one point of this S^1 over any other and the vortices will form. Next the D3-brane will start falling towards the tip due to the brane-antibrane attraction. During this period inflation will take place, the relative separation of the brane-antibrane system will play the

role of the inflaton. This inflationary period will dilute away the DBI vortices. Further, the domains with different values of the complex ϕ field (all belonging to the S^1) that formed on the world volume of the D3-antibrane will become exponentially large during the inflationary epoch. Hence towards the end of inflation, the D3-brane will effectively fall towards one point on the S^1 vacua inside each exponentially large region (which will contain the visible universe). Hence, within each exponentially large region there is no spontaneous symmetry breaking and no formation of vortices. In short, the global vortices will form when the D3-antibrane settles at the tip region. However, these vortices will get diluted during inflationary epoch. There will be no vortex formation when inflation ends and the D3-brane settles at the tip region as within eponentially large regions (the domains on D3-antibrane inflated to exponentially large sizes during inflation) the D3-antibrane sits at a unique point on the S^1 vacua. At this point the usual cosmic string production still happens due to brane-antibrane annihilation [12, 13].

6. Comparison with k-defects

The distinguishing features of DBI global vortex solution are due to the square root kinetic term in the action. A general analysis of the defects arising from a class of actions with a nonlinear kinetic term has been done by Babichev in Refs [16] and [17] - these defects are called the *k-defects*. In these models, the kinetic term of the scalar field has the general form $M^4 K(X/M^4)$, where $X = 1/2(\partial_\mu \phi)^2$ and M is some mass scale (called the *kinetic mass* scale) other than the mass scale η that arises in the potential term $V(\phi) = (\phi^2 - \eta^2)^2$. The size and energy density of the k-defects is set by a combination of M and η which depends on the exact functional form of the kinetic term $K(X/M^4)$. For example (see [16]), for a simple power law $K = -(X/M^4)^\alpha$, the size of the defect is given by the scale $M^{-1}(\eta/M)^{1-2/\alpha}$.

This analysis of k-defects can be translated to our DBI case easily. The kinetic mass scale is simply the warp factor $f(\phi)$. The potential energy $V(\phi)$ should be measured in units of this warp factor. Hence, the effect of the DBI action becomes important only when $f(\phi)V(\phi)$ is order unity. Indeed, in the case of k-defects M supplies an additional mass scale which can combine with η (the GUT scale, say) to give an effectively low tension for the k-defects. This is similar to the observation that in a warped background the effective tension of a cosmic string can be low due to the warp factor $f(\phi)$.

The appearance of the wrinkle for the DBI vortex, however, is a feature that is essentially due to the relativistic nature of the square root kinetic term. In the somewhat different context of tunneling of D-branes [1], the appearance of the wrinkle is explained as due to the Schwinger effect. This effect, therefore, is not seen in the general k-defect analysis of [16].

7. Discussion

We have considered the DBI action for a complex field ψ with a potential term $V(\psi)$ and constructed the vortex solutions which are generalizations of the usual global vortex of the complex scalar field theory. For substringy heights of the potential the vortex resembles

the usual field theory global vortex. For stringy potentials the vortex develops a wrinkle analogous to the instanton wrinkle found in [1] [1]. We have neglected the world volume gauge fields. Including the world volume gauge fields might lead to the DBI analogues of field theory local strings. These vortices should not be confused with the vortex solutions on a tachyon profile that are produced upon tachyon condensation initiated by the brane-antibrane instability. Unlike tachyon vortices, the DBI vortices can form even when the brane-antibrane separation is large in string length units.

A similar effect was seen in the context of DBI instanton in [1] where the instanton interpolating between the metastable and the true vacuum of the potential $V(\phi)$ was studied in the thin wall limit. When the height of the barrier $V_0 < 1$ one gets the usual QFT instanton. However when $V_0 > 1$ the instanton develops a wrinkle due to the double valuedness (and turning around) of the instanton configuration. For the vortex solution the wrinkle appears at $V_0 \approx 0.8$. This difference in the critical V_0 is due to the friction term present in the vortex equation of motion (eq. (2.2)). The wrinkled instanton in [1] was constructed in the thin-wall limit where the friction term is set to zero.

To understand why the wrinkle appears around $V_0 = 0.8$ for the vortex solution, let us consider the DBI instanton solution in a bit detail. As in ref. [1], let $\phi(\rho)$ denote the scalar field as a function of the Euclidean coordinate ρ that leads to the instanton solution. In the absence of any warp factor, the Hamiltonian for this field is given by $H = 1 - (1 - (d\phi/d\rho)^2)^{-\frac{1}{2}} - V(\phi)$. In the thin wall limit, this Hamiltonian is conserved. The zero energy instanton configuration is then obtained by considering $H = 0$ and solving for ρ as a function of ϕ:

$$\rho(\phi) = \int d\phi \frac{1 - V(\phi)}{\sqrt{V(\phi)[2 - V(\phi)]}}. \tag{7.1}$$

When $V(\phi) \ll 1$, the integrand can be approximated as $1/\sqrt{2V(\phi)}$, which leads to the usual field theory instanton for standard kinetic term. However, when $V(\phi)$ can exceed unity, the DBI wrinkle appears. In particular, when $V(\phi) = 1$, then $d\phi/d\rho \to \infty$. This corresponds to the double valuedness of the field configuration as a function of ρ, i.e. the formation of the wrinkle. Hence, for the DBI instanton, the wrinkle corresponds to the double-valuedness of the field and this happens at the threshold value of $V_0 = 1$ in the thin wall limit. Now we can see why for the DBI vortex the wrinkle appears at a lower value of $V_0 \simeq 0.8$. This is because for the vortex solution there is no corresponding "thin wall limit" when the friction term can be neglected. In fact, for the existence of the vortex solution, the friction term (the first derivative of the ψ field) is very important and cannot vanish. Therefore, the Hamiltonian is not a constant any more. This friction term is responsible for decreasing the threshold value when the wrinkle appears to $V_0 \simeq 0.8$.

The appearance of the wrinkle can also be understood by following the analogy with the DBI instanton of [1]. Assuming that the vortex configuration will always have a large enough value of the radial coordinate r (i.e. apriori assuming the formation of a wrinkle),

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[1]Vortex solutions for the DBI action have been studied by various authors including [8–11]. These vortex solutions (BIons) can develop a throat and have double valued ϕ.

the second and the third terms in eq. (4.2) (i.e. $\frac{\gamma}{r}\frac{d\chi}{dr}$ and $\gamma\frac{n^2\chi}{r^2}$) can be neglected. The equation of motion then resembles the thin wall equation of motion for the DBI instanton in [1] and following the reasoning for the instanton, a wrinkle must develop. This is, of course, a heuristic way to see the appearance of a wrinkle in the vortex, which will fail to give the exact value of the radial coordinate r or height of the potential V_0 where the wrinkle appears.

The asymptotics of the DBI vortex at radial infinity will remain the same as that for the usual field theory vortex of section (3). This is because at radial infinity the gradient term vanishes and the DBI kinetic term does not play any role. The DBI effect only occurs near the origin (i.e. near the core of the vortex) where the gradient term is big and sources the DBI term. Since the DBI global vortex asymptotics at radial infinity are the same as the usual field theory global vortex asymptotics at infinity, there will still be a logarithmic divergence in the total energy of the field configuration. However, as explained before, inspite of the double-valuedness of the DBI vortex field configuration, the Hamiltonian density will remain finite everywhere.

In section (5) we considered the possibility of the cosmological formation of these vortices in a brane inflation scenario. Brane inflation consists of two steps. First, before the inflation can begin, a D3-antibrane migrates down the throat and settles at the tip. In the presence of an appropriate moduli space, vortices can form on the antibrane world volume when it settles at the tip. However, in the next stage a D3-brane is attracted to the antibrane and this leads to inflation. The inflationary stage washes away the vortices. Further, the different domains on the antibrane which have the ψ field at different points of the S^1 become exponentially large during inflation. Consequently, the D3-brane effectively falls towards a single point on the S^1. No DBI vortices form at the end of inflation.

Our approach in this note has been phenomenological. Motivated by the results for the D3-brane moduli space on a warped deformed conifold in [7], we consider a potential $V(\psi)$ along the S^3 angular coordinates for the D3-brane and examine the global vortex solution. Whether or not a wrinkle will form depends on the height of the potential. If $V(\psi)$ never attains stringy heights, then there is no possibility of a wrinkle. At this point we simply note that the simplest DBI inflation scenario [3] considers a DBI action of the form eq. (2.1) (with gravity added). The inflaton potential $V(\phi)$ is generated by radiative or bulk effects whose value must be large when measured in terms of the warp factor/local string units $f(\phi)$. In particular, power law inflation occurs at late times (i.e. when $t \to \infty$) as the mobile D3-brane is nearing the tip. The warp factor and the potential have late time dependence $f(\phi) \to t^4$ and $V(\phi) \to 1/t^2$ which leads to $f(\phi)V(\phi) \to t^2$ in the late time DBI regime. In such a setting it appears that one can generate large potentials needed for interesting DBI effects. For the appearance of a wrinkle we require a potential whose value is order one in local string units. Another issue while considering the appearance of the wrinkle is the trustability of the DBI description. The DBI description is valid whenever the extrinsic curvature of the solution $\psi(r)$ is low. In the absence of any warp factor this

extrinsic curvature is given by [1, 14]

$$K(\psi) = \frac{1}{\sqrt{\alpha'}} \frac{\partial V(\psi)}{\partial \psi}, \qquad (7.2)$$

i.e. as long as the slope of the potential is small in string units, the DBI action has small higher derivative corrections.

It would be interesting to see if there is some version of brane inflation on a warped deformed conifold which can lead to the formation of these DBI vortices. Further, it would be interesting to find the local string version of these DBI global vortices as local strings can be realistic cosmic string candidates. If such local strings still develop a wrinkle due to the DBI effect, this wrinkle would be the result of the stringy DBI action. Such cosmic strings would, in principle, differ from their field theory cousins and perhaps lead to novel phenomenological consequences.

Further one could construct DBI defects, that are extensions of the DBI vortices we have studied, on the world volume of multiple D3-branes. The theory would then be a non-abelian one. One could then look , for example, for the DBI extension of the t' Hooft-Polyakov monopole. It is tempting to speculate that just as what we saw for the DBI vortex solution, there will be no DBI effects present at radial infinity of the monopole solution. This is because the field gradient vanishes at large distances from the core of the monopole. The DBI effect would occur for large field gradients near the core of the monopole and lead to the formation of a wrinkle near the core for potentials with stringy heights.

Acknowledgments

A. Numerical calculation and the boundary conditions

The plots for the vortex configuration (χ versus r) were obtained by numerically solving for the equation of motion for the scalar field modulus χ given in eq. (4.2). The boundary conditions are given by $\chi(r = \infty) = 1$, and $\chi(r = 0) = 1$. While this is straightforward to do for small values of V_0, once V_0 exceeds 0.8, the field configuration becomes double valued and this numerical method for obtaining the solution fails. To get around this problem appearing due to the double-valuedness of χ, we rewrite the equation of motion with the radial coordinate r as a function of the field modulus χ, i.e. $r(\chi)$. One can do this by starting with the action in eq. (2.2). With the ansatz $\psi(r, \theta) = e^{in\theta} \chi(r)$, we get

$$S = -2\pi \int dr r \left[\sqrt{1 + \left(\frac{d\chi(r)}{dr} \right)^2 + \frac{n^2 \chi(r)^2}{r^2}} - 1 + V(\chi) \right]. \qquad (A.1)$$

The equation of motion obtained from this action can be solved to obtain χ as a function of r. However, we can rewrite the action as follows to have r as a function of χ

$$S = -2\pi \int d\chi\, r(\chi) \left[\sqrt{1 + \left(\frac{dr(\chi)}{d\chi}\right)^2 \left[1 + \frac{n^2\chi^2}{r(\chi)^2}\right]} + (V(\chi) - 1)\frac{dr(\chi)}{d\chi} \right], \quad \text{(A.2)}$$

whose variation leads to the equation of motion for r as a function of χ

$$r(\chi)\frac{d}{d\chi}\left[\frac{\left(1 + \frac{n^2\chi^2}{r(\chi)^2}\right)}{\sqrt{1 + \left(1 + \frac{n^2\chi^2}{r(\chi)^2}\right)\left(\frac{dr(\chi)}{d\chi}\right)^2}}\frac{dr(\chi)}{d\chi} \right] + \frac{\left(\frac{n^2\chi^2}{r(\chi)^2}\left(\frac{dr(\chi)}{d\chi}\right)^2 - 1\right)}{\sqrt{1 + \left(1 + \frac{n^2\chi^2}{r(\chi)^2}\right)\left(\frac{dr(\chi)}{d\chi}\right)^2}} \quad \text{(A.3)}$$
$$+ r(\chi)\frac{dV(\chi)}{d\chi} = 0.$$

This equation can be solved numerically. The plots shown in figures 1 to 8 were obtained by numerically solving this equation subject to the boundary conditions $r(\chi = 1) = \infty$, and $dr(\chi = 1)/d\chi = \infty$ (where ∞ was replaced by sufficiently large numbers in the mathematica numerical calculation).

A few words about the boundary conditions. In the usual coordinates (χ as a function of r) the boundary conditions are taken to be $\chi(r = \infty) = 1$ and $\chi(r = 0) = 0$. This translates to $r(\chi = 1) = \infty$ and $r(\chi = 0) = 0$. However, this leads to a problem for the DBI case. For large values of the potential (roughly $V_0 \geq 0.8$) χ becomes double valued as a function of r (although $r(\chi)$ still remains single valued as a function of χ) and the field configuration never reaches $r = 0$. Instead, the configuration terminates at some finite value of r at $\chi = 0$. The boundary conditions which are appropriate to describe this behavior are given by $r(\chi = 1) = \infty$ and $dr(\chi = 1)/d\chi = \infty$. These two boundary conditions give the usual QFT vortex solutions for $V_0 \ll 0.8$ where the field configuration sits at the vacuum at radial infinity and sits at the top of the potential barrier at radial origin. However for large barriers ($V_0 \geq 0.8$) the "wrinkle" develops and the field configuration starting at the vacuum at radial infinity never quite makes it to the radial origin.

Qualitatively one can see why this happens. Let us go back to the usual description of the field where we consider χ as a function of r. From the equation of motion eq. (4.2), regarding r as a time coordinate, we see that χ is moving in the inverted mexican hat potential. Our boundary conditions ($r(\chi = 1) = \infty$ and $dr(\chi = 1)/d\chi = \infty$) correspond to field sitting at a maximum of the inverted mexican hat at $\chi = 1$ at $r = \infty$ and then rolling down towards the minimum of the inverted potential at $\chi = 0$. As the field rolls down and approaches the origin at $\chi = 0$, the relativistic factor γ decreases. At some point, the speed becomes infinite and $d\chi/dr$ becomes double valued. The energy density of the configuration is still finite. This is because we are really looking at a static solution and our γ is a static analogue of the Lorentzian γ factor. For brane dynamics (as discussed in [3]), the Lorentzian γ factor diverges as the brane speed approaches the speed of light and, consequently, the Hamiltonian also diverges. However in our case, when the "speed" $d\chi/dr$ diverges, γ vanishes. Consequently there is no divergence in the Hamiltonian density.

One could ask what would have happened if we had tried to solve numerically the equation of motion eq. (4.2) by specifying the two boundary conditions at the origin (i.e. at $r = 0$ considering $\chi(r)$ as a function of r). One can certainly specify the boundary conditions in this way: start with $\chi(r = 0) = 0$ and $d\chi(r = 0)/dr = c$ where c is a positive number. One can slowly increase c from zero. Note that this corresponds to launching the field modulus χ at speed c starting from the bottom of the inverted potential at $\chi = 0$ towards the local maximum at $\chi = 1$. We would like χ to reach this maximum with zero speed at $r = \infty$ and sit at this unstable maximum. For $V_0 < 0.8$, increasing c from zero upwards, one eventually finds the value of c which leads to this field configuration which corresponds to the vortex solution. However, once $V_0 = 0.8$ (approximately) no finite value of c is large enough to take χ to the maximum of the inverted potential. This happens due to the relativistic effect which manifests as the friction term in the equation of motion. The infinite value of c for $V_0 = 0.8$ implies that the solution $\chi(r)$ is about to become double valued as a function of r. For $V_0 > 0.8$ no solutions exist satisfy the boundary conditions $\chi(r = 0) = 0$ and $\chi(r = \infty) = 1$. Solutions exist where the field χ sits at the vacuum configuration (i.e. $\chi = 1$) at radial infinity with zero energy density at infinity (i.e. $d\chi(r = \infty)/dr = 0$). But these solutions never interpolate all the way to $r = 0$. Furthermore these solutions are double valued in χ and we have to switch to eq. (A.3) to be able to construct these numerically.

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